Presented by Dorukhan Ardag
Co-Author: Donald T. Resio
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Development of source terms for coupled modeling and remote sensing applications. Part I: Nonlinear source term

UNF UNIVERSITY of NORTH FLORIDA
College of Computing, Engineering & Construction

Motivation

- The initial motivation for transitioning from 2G models to 3G models was that 1) the 2G models could not represent the nonlinear source term within its parameterizations and 2) the 2G models would have to be locally tuned for optimal performance.
- Today's 3G models are tuned holistically for optimal global performance; however, this does not ensure local optimality. Consequently, the need for tuning coefficients has not diminished.
- Holistic tuning of 3G models has focused on comparisons to integrated spectral parameters (wave height, peak wave period and mean wave period). Comparisons to spectral shape have been subjective; however, recently a set of metrics for spectral comparisons has been proposed.
- These metrics appear to reinforce the concept proposed by Hasselmann that nonlinear interactions control many aspects of spectral shape.
- Here we examine the application of the Two Scale Approximation (TSA) to produce accurate spectral shapes and reliable detailed balance among source terms as required for accurate applications to couple modeling and remote sensing. (As shown by Yalin Fan on Monday)

Overview

Our main goal is to build a model basis which can replicate the observed spectral metrics shown in Resio, Vincent and Ardag (2015).

- Introduce spectral metrics and basic TSA approach.
- Describe recent progress in the modified TSA.
- Perform evolution tests for the modified TSA.
- Compare the operational speed of the TSA with the Discrete Interaction Approximation (DIA).
- Conclusions

Spectral Metrics

- 1. An equilibrium range with an extent that depends on wave age,
- 2. A spectral peakedness defined in an context which depends on wave age,
- 3. An equilibrium range coefficient which is consistent with the momentum balance entering the wave field and passing through the equilibrium range,
- 4. A transition from to form at a location within the spectrum which varies as a function of wave age,
- 5. A relaxation from a perturbation that returns the spectrum to an appropriate equilibrium shape,
- 6. An evolution of a spectrum beyond the limit at which a fully-developed wave height is achieved, and
- 7. A bimodal directional distribution with the lobe angles and lobe ratios consistent with observations from spatio-temporal observations.

Coupled Models need Accurate Spectra/Detailed Balance

Transfer Function* (Webb, 1978) *:

$$T(\vec{k}_1,\vec{k}_3) = 2 \oint D(\vec{k}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4)\; C(\vec{k}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4) \left|\frac{\partial W}{\partial n}\right|^{-1} ds$$

D is a function consisting of triplets of action densities:

$$D(\vec{k}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4) = n(\vec{k}_1)\, n(\vec{k}_3)\, [n(\vec{k}_4) - n(\vec{k}_2)] + n(\vec{k}_2)\, n(\vec{k}_4)[n(\vec{k}_3) - n(\vec{k}_1)]$$

The TSA approach (Resio and Perrie 2008) :

$$n_i = \hat{n}_i + n_i^{`}$$

Parametric (Broad Scale) Residual (Local Scale)

Two Scale Approximation (TSA)

Source term becomes:

$$S_{nl}(f,\theta) = B + L + X$$

Pre-computed by using Full Boltzmann Integral (FBI)

New equation:

$$\frac{\partial n_1}{\partial t} = B + \iint \oint N_*^3 \, C \left| \frac{\partial W}{\partial n} \right|^{-1} ds\, dk_3\, k_3\, d\theta$$

Approximations:

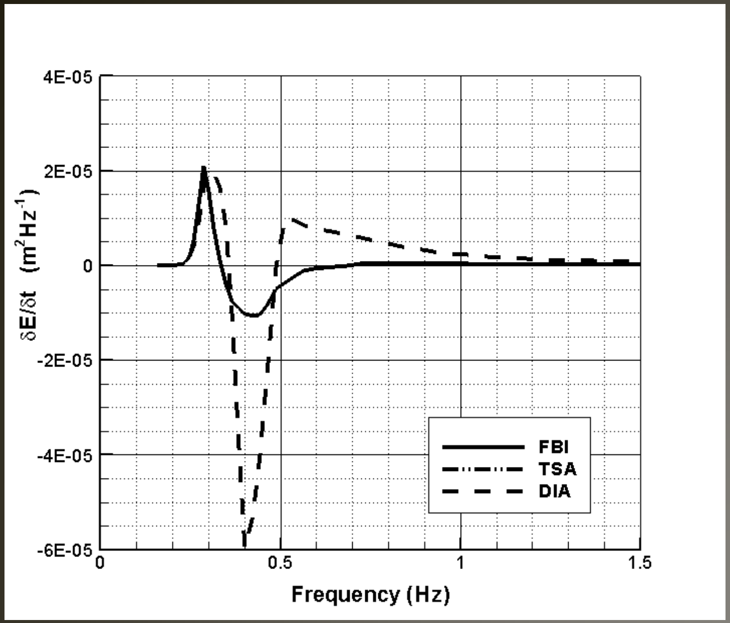
Action density triplets

$$N_*^3 = \hat{n}_2\, \hat{n}_4 (\grave{n}_3 - \grave{n}_1) + \grave{n}_1 \grave{n}_3 (\hat{n}_4 - \hat{n}_2) + \hat{n}_1 \grave{n}_3 (\hat{n}_4 - \hat{n}_2) + \hat{n}_3 \grave{n}_1 (\hat{n}_4 - \hat{n}_2)$$

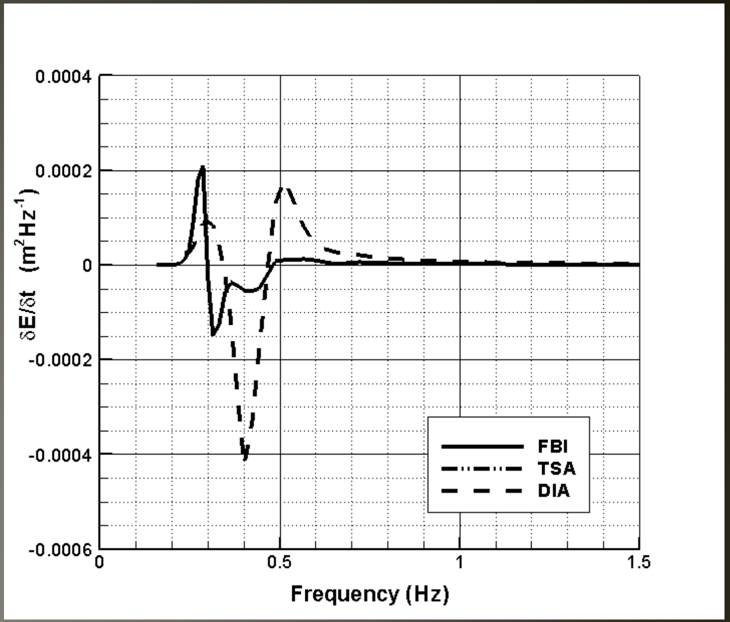
$$\Lambda_p = \oint C \left| \frac{\partial W}{\partial n} \right|^{-1} (\hat{n}_4 - \hat{n}_2)\, ds \Delta\theta$$

$$\Lambda_d = \oint C \left| \frac{\partial W}{\partial n} \right|^{-1} (\hat{n}_4 \hat{n}_2)\, ds\, \Delta\theta$$

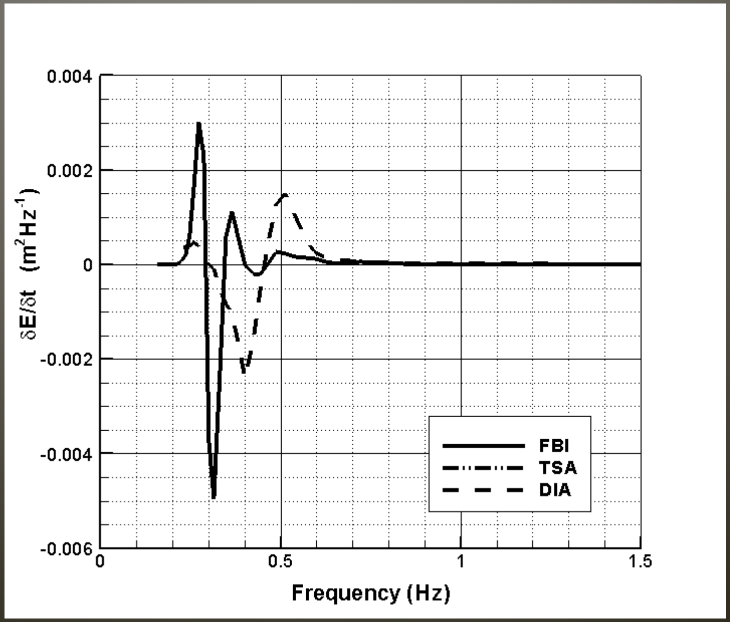
$\gamma_r = 1.02$



$\gamma_r = 2.04$



$\gamma_r = 4$



Accuracy of a Broad Scale only Model

- Broad Scale only models have been shown to be applicable with no tuning/limiting factor. Hanson et al. (2009) applied this approach on Pacific scale and received relatively accurate results. (WAVAD/WISWAVE is based on the Resio and Perrie (1989) 1-parameters Snl – essentially a Broad Scale-only model with no basin-specific tuning)

- We believe that ,for slowly varying winds, this approach can increase the efficiency of a model significantly, running it on Broad Scale a relatively high percentage of the time.

- The initial TSA was operating with the same broad scale terms for evaluating conditions.

TABLE 3. Significant wave height performance summary (WAVEWATCH III: WW III).

	Wave height performance scores					
	Temporal correlations			Quantile–quantile		
Component	WAM	WW III	WAVAD	WAM	WW III	WAVAD
Wind sea	0.79	0.88	0.83	0.82	0.92	0.88
Young swell	0.84	0.85	0.79	0.90	0.89	0.86
Mature swell	0.72	0.78	0.73	0.78	0.83	0.81
Combined	0.79	0.84	0.78	0.83	0.88	0.85

Computational Speed

- Broad scale is pre-computed, Local scale:

Pre-computed matrices

$$\sum_{i=1}^{N_k}\sum_{j=1}^{N_\theta}\{(\grave{n}_3 - \grave{n}_1)\Lambda_d + (\grave{n}_1\grave{n}_3 + \hat{n}_1\grave{n}_3 + \hat{n}_3\grave{n}_1)\Lambda_p\}k_3\Delta\bar{k}$$

Initial values(Resio and Perrie 2008):

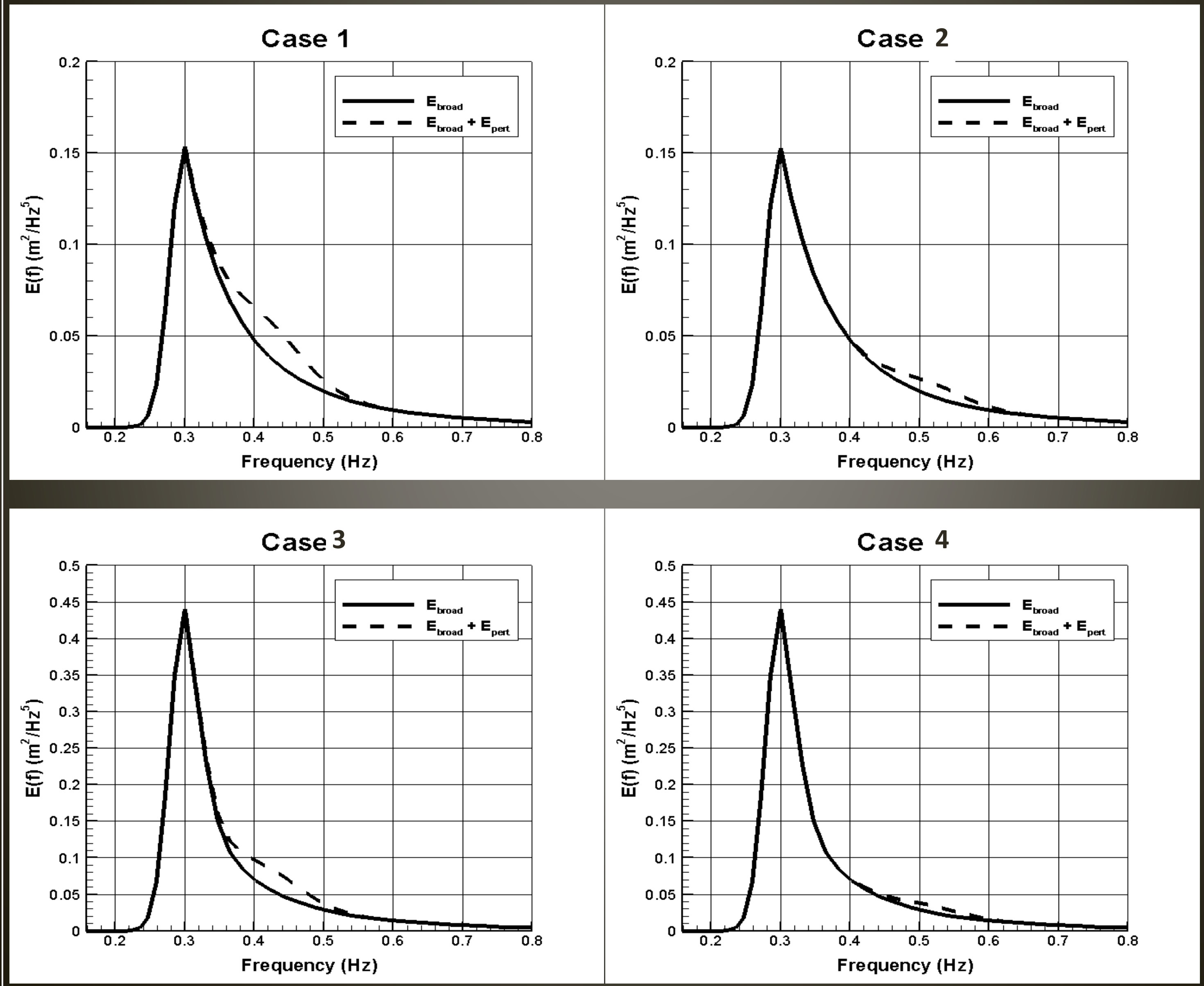
$N_k = N_f = 11$

$N_\theta = 17$

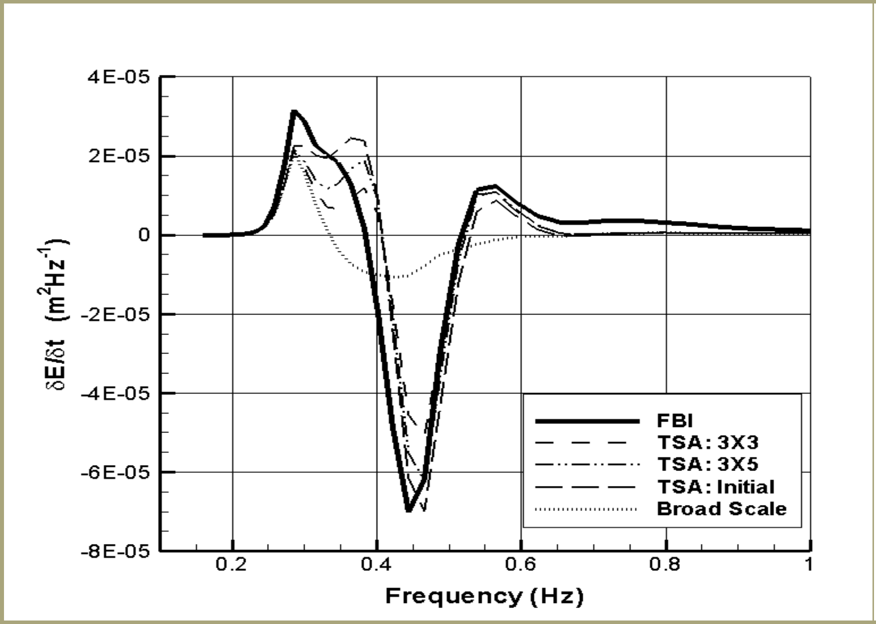
Input Spectra for Initial Testing

Cases	Relative Peakedness	Frequency Location of the Perturbation (Hz)
1	1.02	0.422
2	1.02	0.513
3	2.04	0.422
4	2.04	0.513

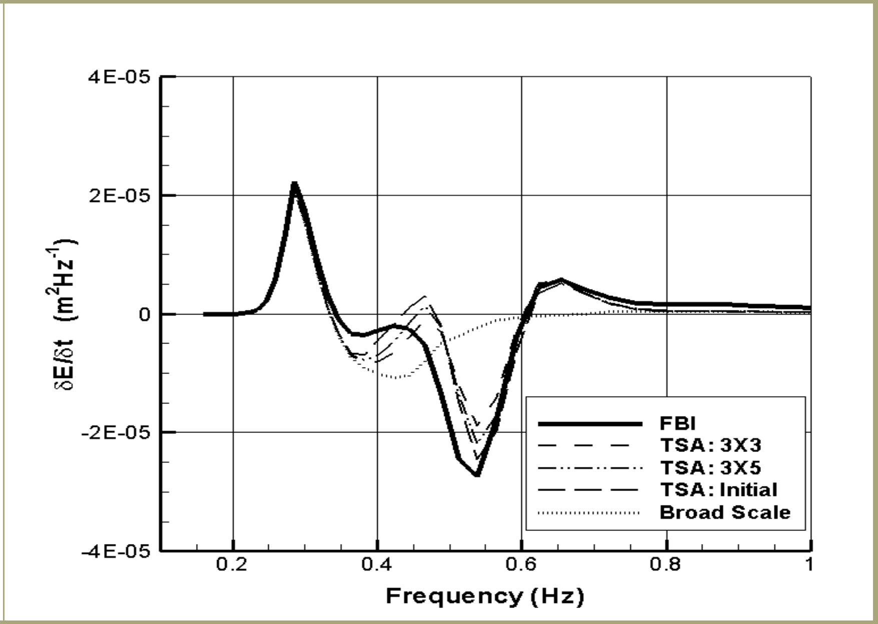
Input Spectrum for Initial Testing-uncompensated form



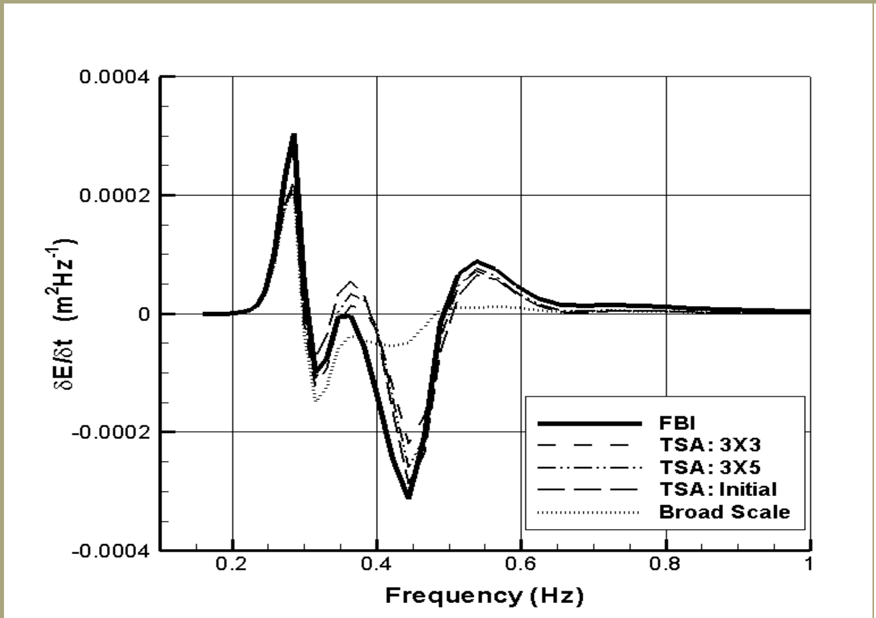
Case 1



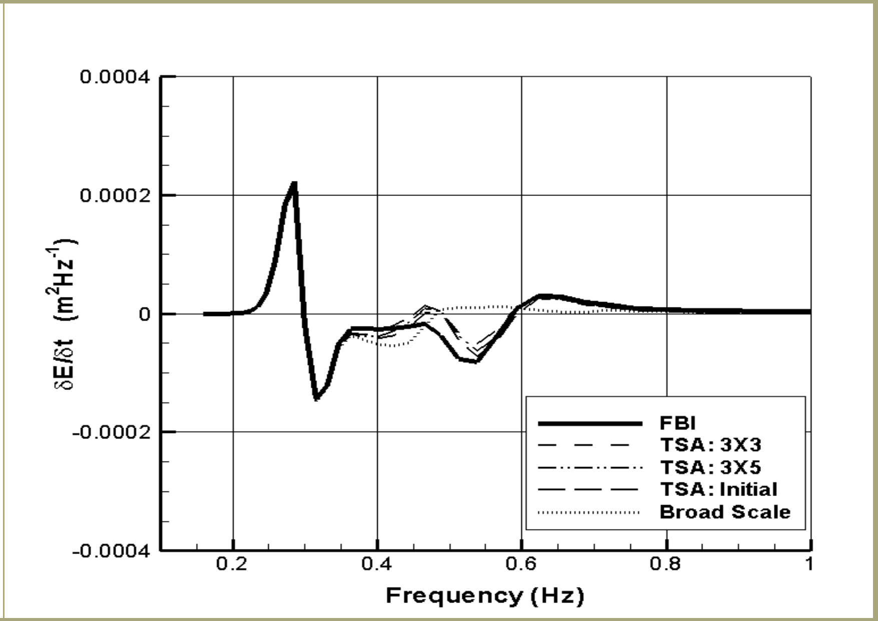
Case 2



Case 3



Case 4



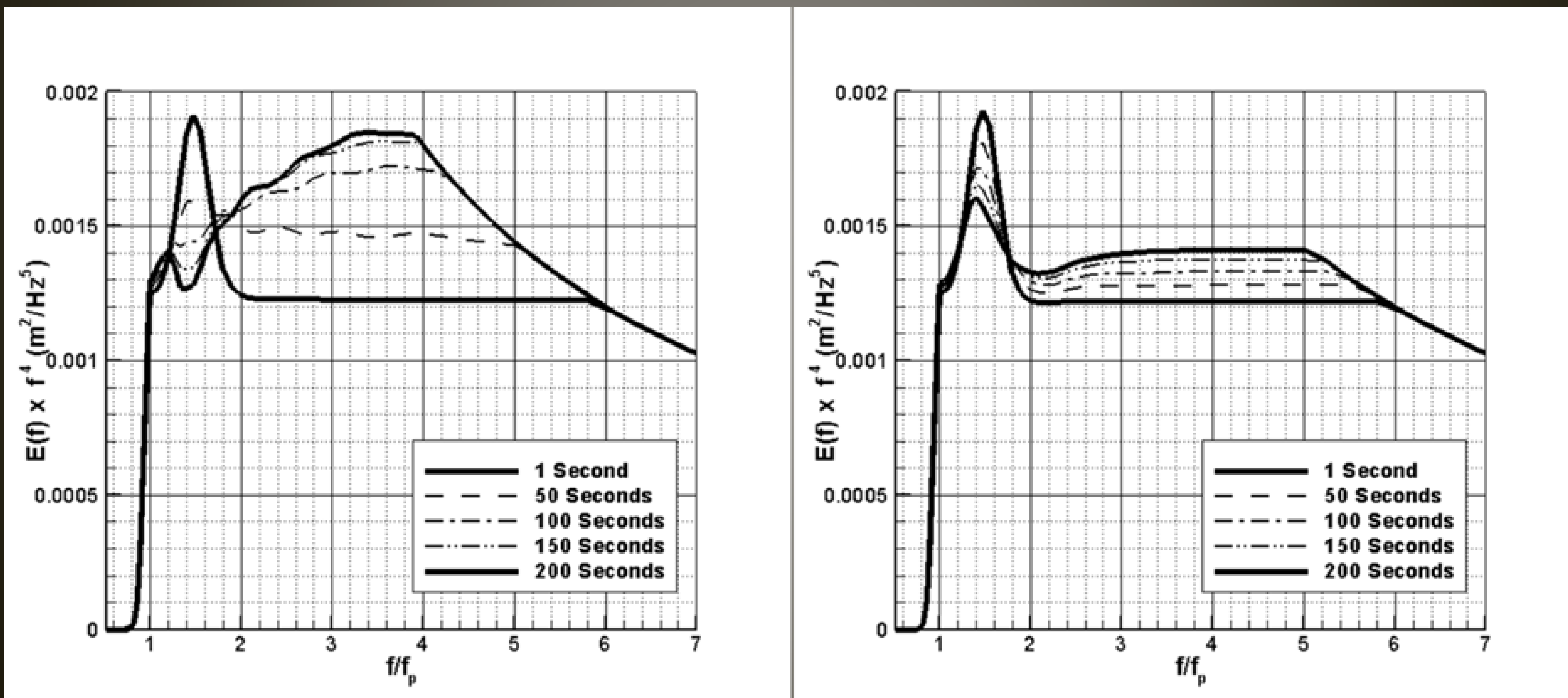
Local Scale

Table of Relative Errors

$$\frac{N_{baseline}}{N_{reduced}} = \frac{17\times 11}{5\times 3} \approx 12.5$$

Local Scale Domain	Perturbation on 21th frequency ring		Perturbation on 25th frequency ring	
	Case 1	Case 3	Case 2	Case 4
3 Frequency & 3 Angle Bands	9%	12%	8%	4%
3 Frequency & 5 Angle Bands	8%	11%	7%	4%
4 Frequency & 3 Angle Bands	8%	11%	7%	4%
4 Frequency & 5 Angle Bands	7%	10%	6%	3%
5 Frequency & 3 Angle Bands	8%	10%	6%	3%
5 Frequency & 5 Angle Bands	7%	10%	5%	3%
Initial Scale	8%	11%	6%	3%
DIA	66%	106%	46%	32%

- The FBI (right panel) and the TSA transition toward an f^{-4} equilibrium form
- The DIA (left pane) transitions toward an $f^{-11/3}$ form.

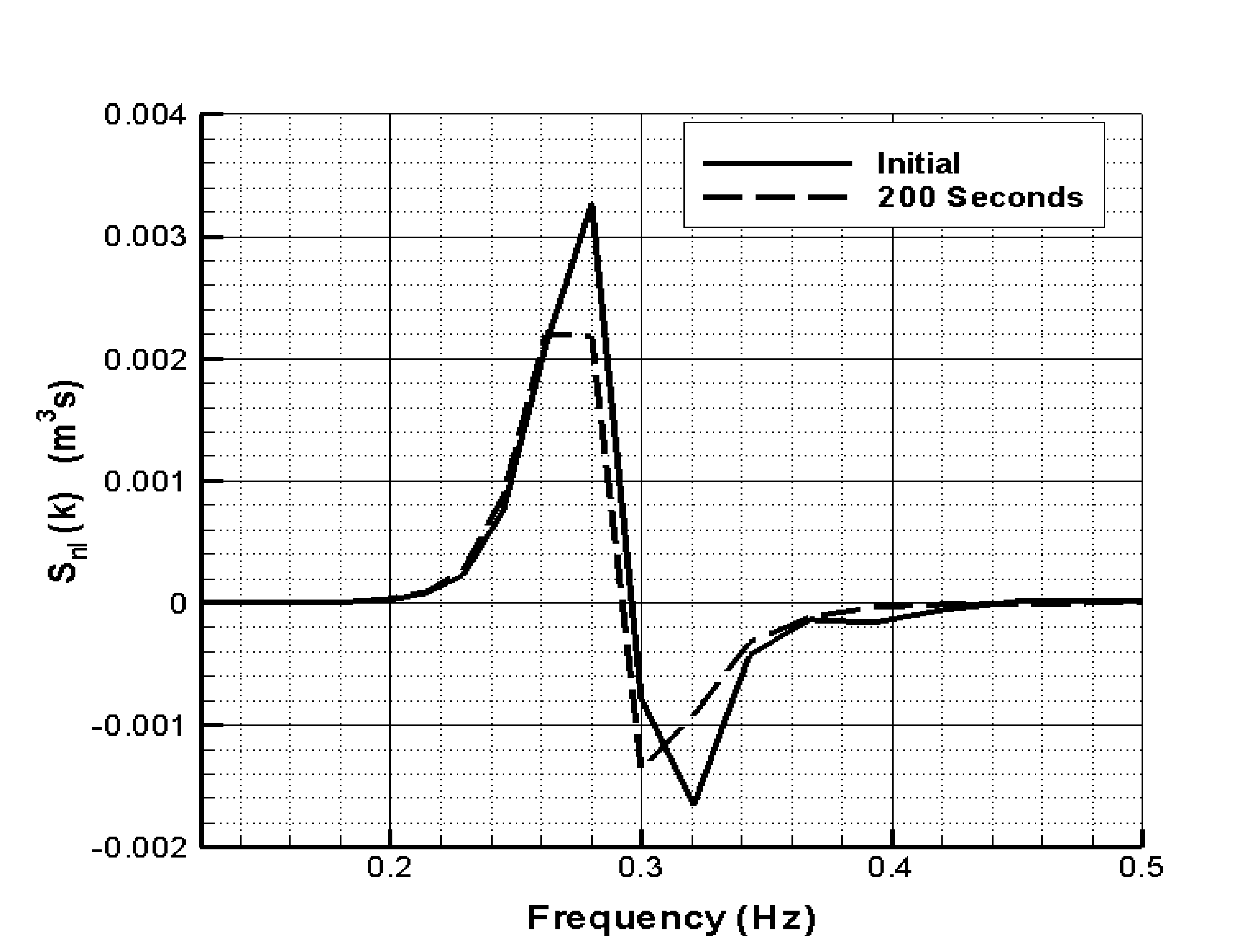


Problems with the Current Approach

- “Flat top” pattern occurred due to frequency discretization.
- Stability was difficult to maintain at high frequencies.
- The original TSA was validated only for snapshots. Not evolutionary tests.

Problems with the Current Approach

“Flat top” pattern occurring due to discrete grids.



Spectral Resolution

- Using 72 angle bands instead of 36 affected the accuracy by decreasing the size of the angle increments by half and thus obliging us to use additional angle bins.

- Among lambda values of 1.03, 1.05 and 1.07, a major dissimilarity was not observed. As long as f_p fell on the discrete frequency grids, lambda of 1.07 performed sufficiently enough.

- Increasing the resolution was accomplished in the "active" discrete element, in other words the discrete space which the f_p is shifting at by introducing sub-increments. After some tests we decided to work with 5 sub-increments.

Parametric Tail based on Irisov and Voronovich (2010)

The f^{-4} equilibrium zone switches to a parametric f^{-5} form at a prescribed f_{ti} number representing a transition point relative to the peak frequency f_p.

$$\frac{2\beta g^{3/2}}{(2\pi)^3} E(f,\theta) f^{-4} = \alpha E(f,\theta) f^{-5}$$

For simplicity in our tests we operated with an f_{ti} of:

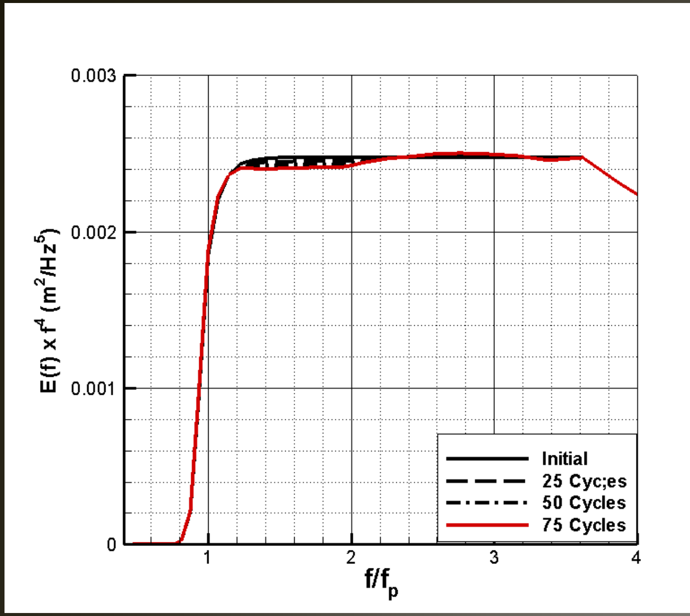
$$f_{ti} = 3.5 f / f_p$$

TSA for Time-stepping Cases

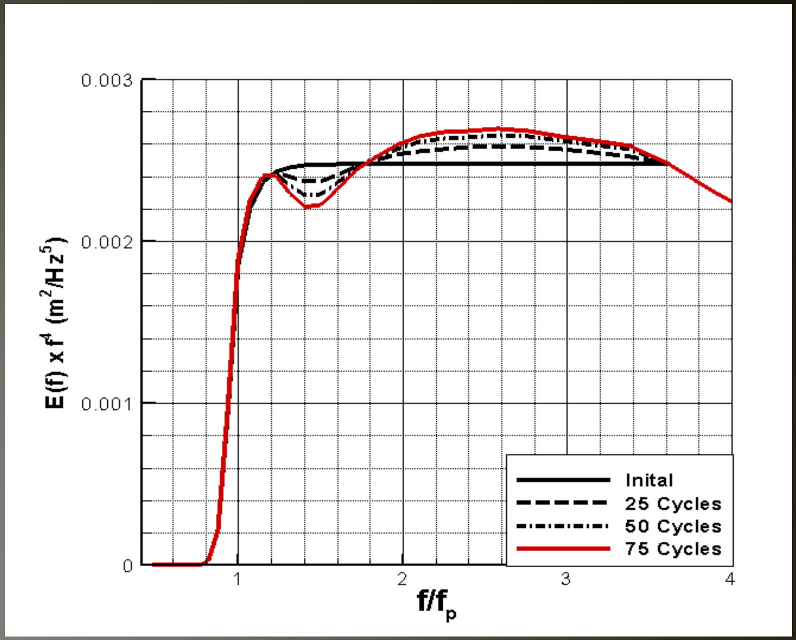
- In our modified approach, we re-parametrize at the end of each time step.
- So we are going to show 3 different cases;

Cases	Peak Frequency (Hz)	Relative Peakedness (γ_r)	Frequency Location of the Perturbation (Hz)
1	0.1	0.75	-
2	0.2	1.4	0.321
3	0.3	2.4	0.481

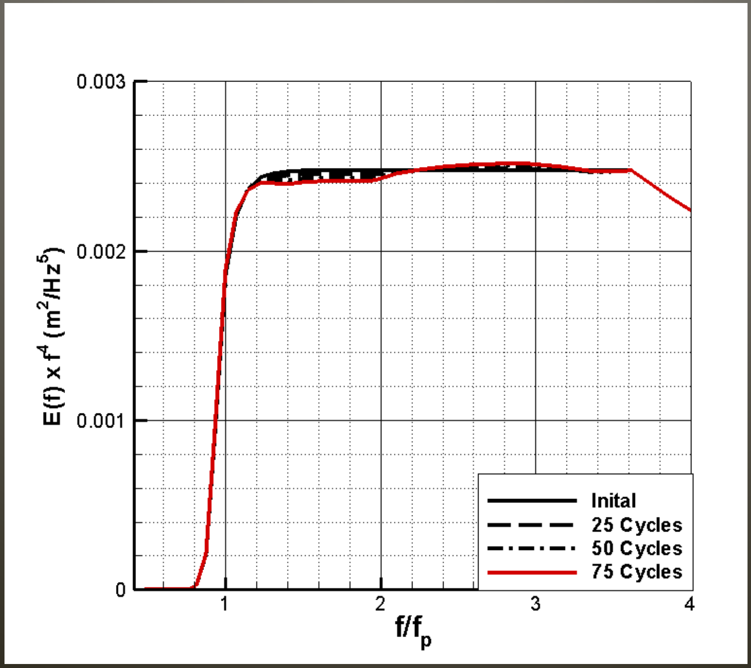
FBI



DIA



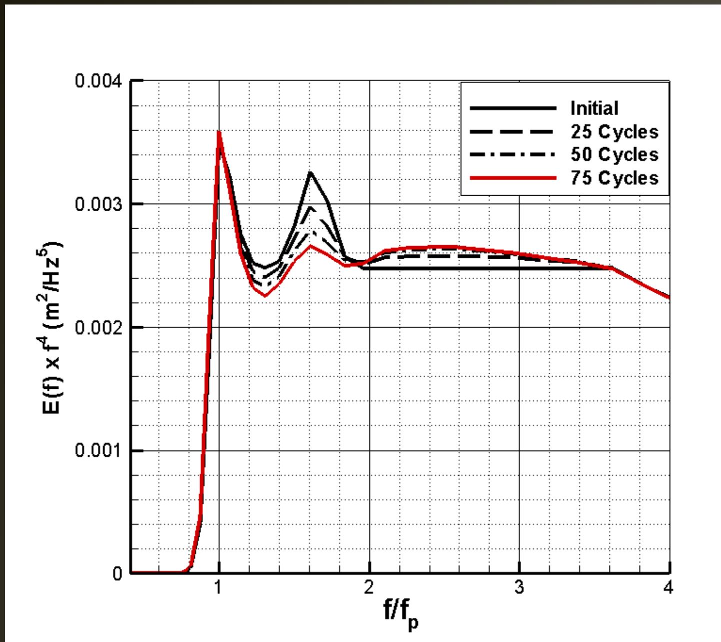
TSA



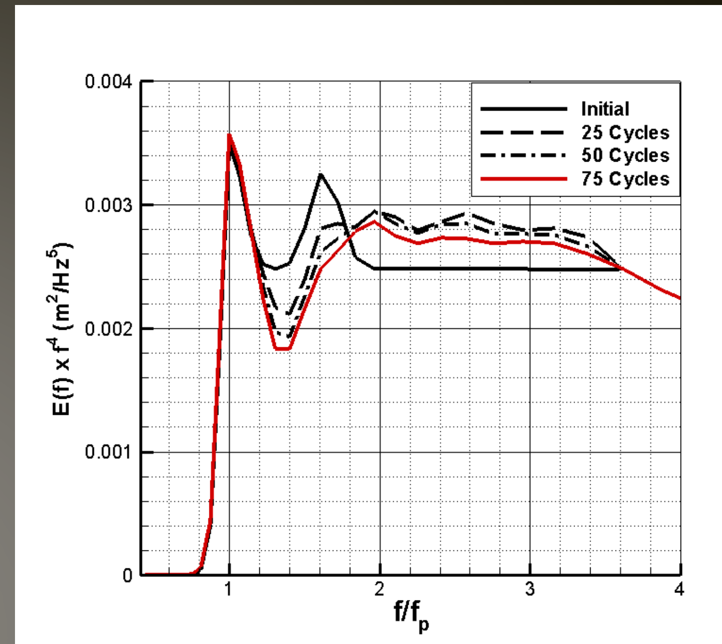
DIA: 35.19 Seconds

TSA: 68.80 Seconds

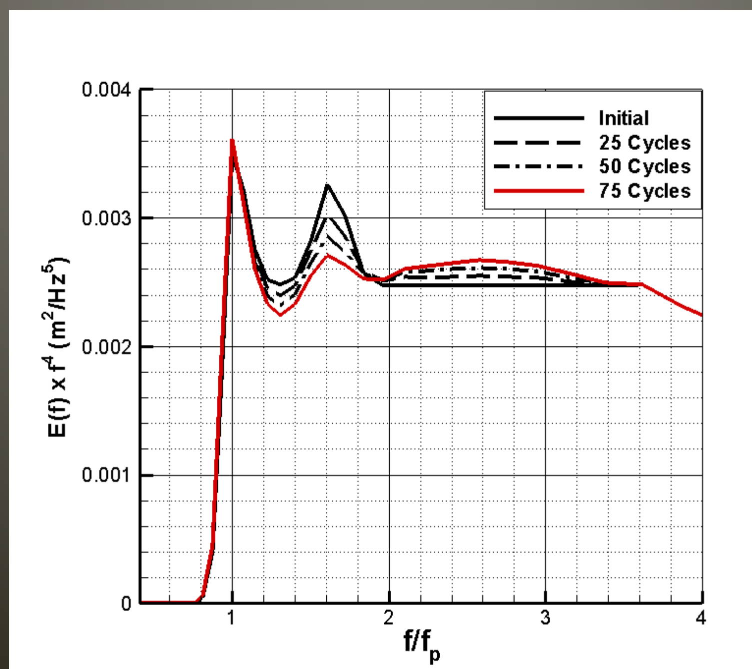
FBI



DIA



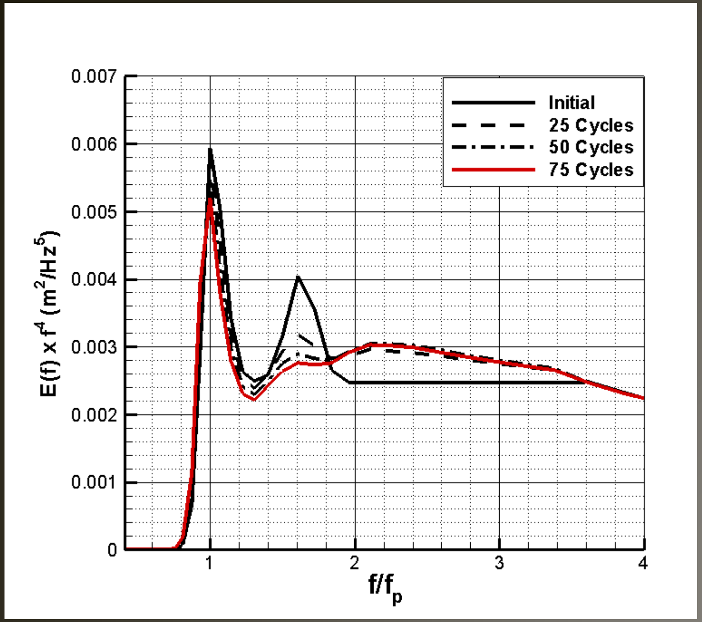
TSA



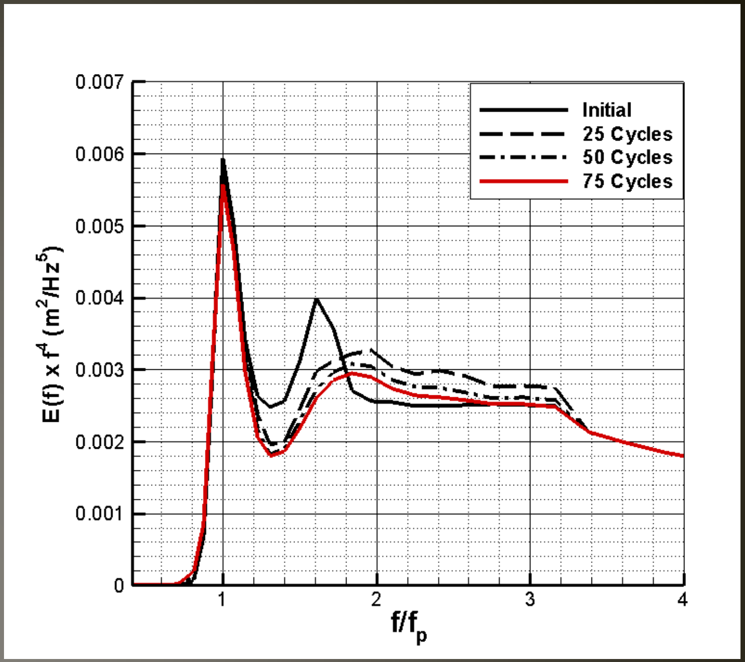
DIA: 16.33 Seconds

TSA: 33.67 Seconds

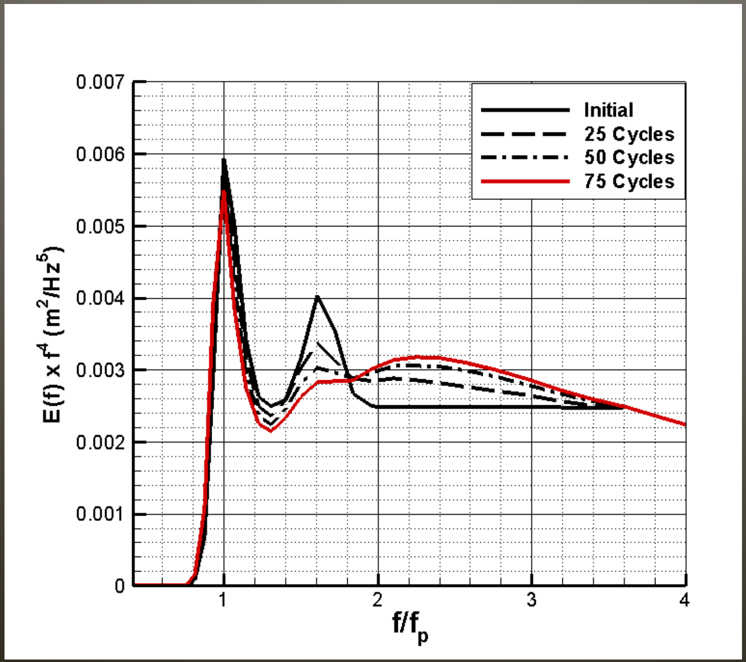
FBI



DIA



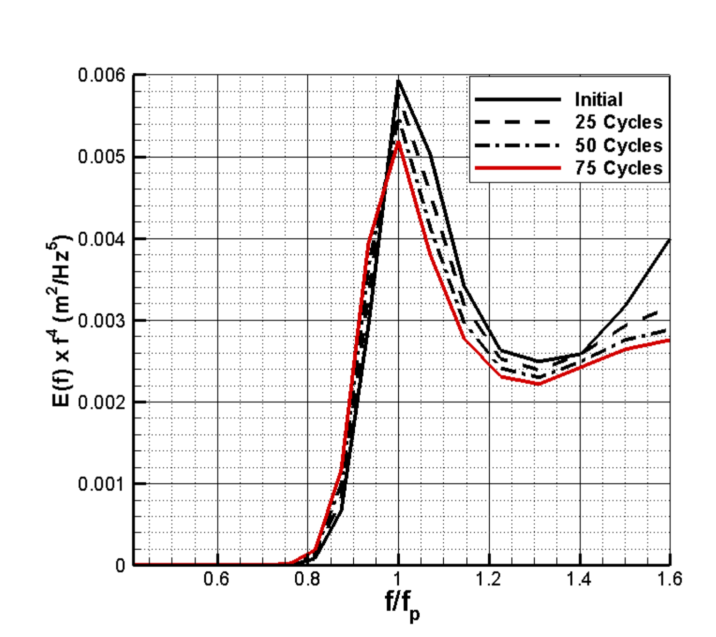
TSA



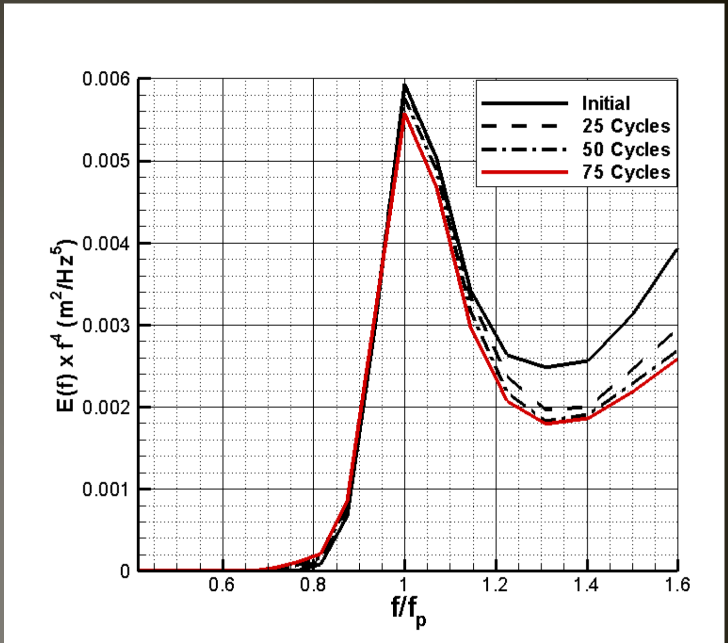
DIA: 9.99 Seconds

TSA: 19.23 Seconds

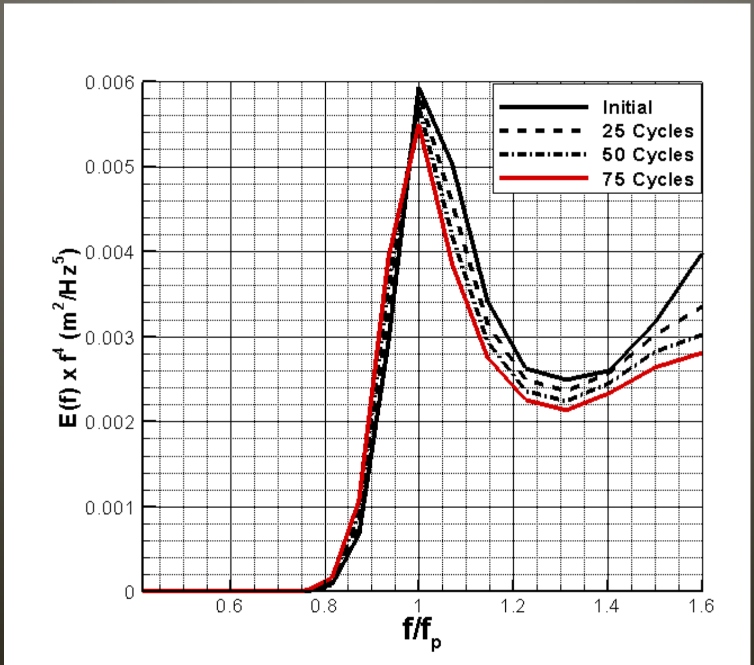
FBI



DIA



TSA



DIA: 9.99 Seconds

TSA: 19.23 Seconds

Conclusion

- The modified TSA has a reduced window size which allows it to be as efficient as the DIA, sometimes even faster. With the modified form the TSA performed well over time evolution tests compared to the DIA.

- Because of this the TSA will provide an improved basis for coupled models and remote sensing.

Future Work

-Operating the TSA on a Local Scale Domain that adds or skips some of the wave number space in order to increase efficiency and accuracy even more.

-Operationalizing the modified TSA with appropriate parametrizations to use it with other source terms.